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Simulation of mercury porosimetry on correlated grids: Evidence for extended correlated heterogeneity at the pore scale in rocks

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A modification of invasion percolation is used to simulate rate-controlled mercury injection experiments on porous media displaying both uncorrelated and correlated disorder. The correlations are generated by a fractional Brownian motion with a cutoff. The introduction of correlated heterogeneity has a marked effect on the behavior of the capillary pressure curve and accounts quantitatively for the features of the experimental curves for sedimentary rock samples. This result suggests that correlated heterogeneity, which is common in porous rock at reservoir scales, persists down to the pore scale. It casts doubt on the use of network models with uncorrelated disorder and classical percolation concepts to model flow behavior at the pore scale in sedimentary rocks. [S1063-651X(98)50112-2]

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The study of flow phenomena in sedimentary rocks has been of great interest to many disciplines for a long time [1–4]. The transport of contaminants in soils and aquifers, and the production of oil and gas from underground reservoirs all depend on the detailed structure or morphology of the pore space. Any theoretical analysis of transport phenomena in such systems requires a realistic description of the pore-scale microstructure. Mercury porosimetry, the forced intrusion of mercury into a porous material, is one of the most widely used methods for the characterization of pore-space morphology.

Capillary pressure curves, measured on rock samples, are routinely used to obtain microstructural information about the rocks. Sample volumes measured are usually on the order of $\simeq 1$ cm^3. At this scale it is assumed that the porous medium is statistically homogeneous, i.e., only small fluctuations in properties occur as a function of sample volume. Theoretical studies of capillary pressure measurements therefore either assume that the pore space is spatially random [5–7] or include spatial correlation at a local scale; for example, between pore body and pore throat sizes [8,9]. Although it is widely recognized that megascopic properties in geological formations (at field scale) are strongly correlated [10–12], it is still unclear whether this behavior persists at smaller scales. This Rapid Communication will focus on this issue.

In order to investigate this issue we use a modification of invasion percolation [13] to simulate rate-controlled mercury injection experiments [14] on porous media displaying both uncorrelated and correlated disorder. We show that the introduction of correlations has a marked effect on the nature of the capillary pressure curve. Moreover, we show that it is only possible to account for the behavior of the experimental data for sedimentary rocks by *including* correlated heterogeneity. This result suggests that correlated heterogeneity, which is clearly evident in porous rock at reservoir scales [10,11], also persists at the pore scale, thus casting doubt on the use of random percolation concepts [15] for modeling flow behavior at the pore scale in porous rocks.

Invasion percolation (IP) [13] is a simple model for describing penetration of a porous medium by a nonwetting fluid, e.g., mercury, when viscous forces can be neglected, and the process is solely governed by the capillary forces. When mercury is injected into a pore, a capillary pressure p_c must be overcome. For a circular throat of radius r and interfacial tension γ, $p_c = 2\gamma\cos\theta/r$, where θ is the contact angle for mercury. Thus, the mercury will preferentially invade the largest pores.

Rate-controlled mercury injection experiments provide far more information on the statistical nature of pore structure than conventional porosimetry [14]. Fluid intrusion under conditions of constant-rate injection leads to a sequence of jumps in capillary pressure which are associated with regions of low capillarity [14]. While the envelope of the curve is the classic pressure controlled curve, the invasion into regions of low capillarity adds discrete jumps onto this envelope. In Fig. 1(a) we show an example of a capillary pressure curve obtained in our laboratories for Berea sandstone under rate-controlled conditions [16]. The detailed geometry of the jumps in the capillary pressure curve over different saturation ranges is shown in Figs. 1(b)–(d).

This is naturally mapped onto the IP model. In the conventional IP, the porous medium is represented by a lattice of sites and bonds. Each site or bond i is assigned a random number r_i which represents an effective size for a pore body or a pore throat located at i. Initially all sites are considered empty. The invading fluid is injected into the medium and fills the pores at each time step by identifying all the sites (or bonds) on the interface between the invading fluid and the empty portion of the pore space, and occupying the site (or

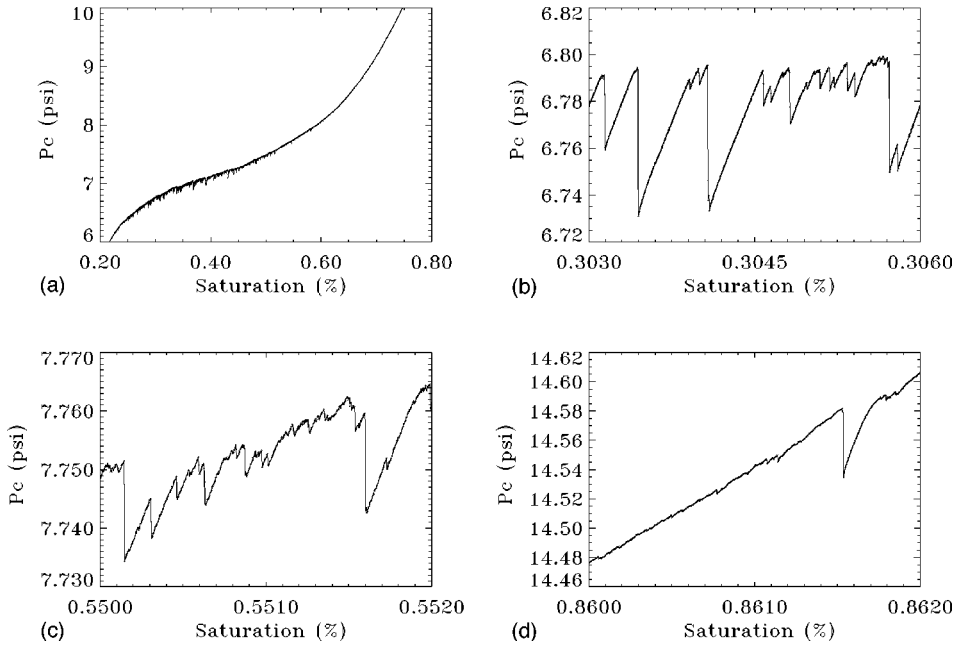

FIG. 1. Experimental constant volume porosimetry curves for Berea sandstone: (a) over large saturation range; (b)–(d) detailed curves over different saturation ranges.

bond) with the largest r_i. As there is no defending phase in mercury porosimetry, we consider IP without a trapping mechanism.

IP models of capillary pressure have previously been used to model the constant-*pressure* curve alone [1,2]. However, since IP models a quasistatic process, one can naturally recover the full constant-volume porosimetry curve from a simulation. We model constant-volume porosimetry on both random and correlated grids. The correlated fields are based on a fractional Brownian motion (FBM), originally used by Hewett [10] for describing reservoir heterogeneity at a large (field) scale. A feature of a network of pores with a statistical description based on a FBM is that it contains long-ranged correlations in pore sizes, with the variance of the pore size given by $\langle r(\mathbf{x})-r(\mathbf{x_0})\rangle = C_0|\mathbf{x}-\mathbf{x_0}|^{2H}$, where C_0 is a constant and the type and extent of the correlations can be tuned by varying H [17]. This contrasts with random grids where adjacent pore sizes are independent. The FBM has not been used or tested for representing the correlations at the pore scale, and we are not aware of any experimental evidence that supports the introduction of correlated heterogeneity extending to the full grid size (which is generated by the FBM). We therefore introduce a cutoff length scale ℓ_c where $\langle r(\mathbf{x})-r(\mathbf{x_0})\rangle = C_0|\mathbf{x}-\mathbf{x_0}|^{2H}$: $|\mathbf{x}-\mathbf{x_0}|<\ell_c$ and $\langle r(\mathbf{x})-r(\mathbf{x_0})\rangle = C_0|\ell_c|^{2H}$: $|\mathbf{x}-\mathbf{x_0}|>\ell_c$. The introduction of ℓ_c allows us to choose the appropriate length scale of the correlations at the pore scale.

The conventional IP algorithm requires minimal modifications to realistically mimic a capillary pressure experiment. The geometry of the conventional IP algorithm considers invasion from one face of the lattice, with a defending phase exiting from the opposite face. In mercury porosimetry the geometry of the displacement is altered. The core is placed in a cell and the mercury completely surrounds the sample. To mimic this process we allow the invader to enter the pore space from all sides.

The volume of a sample studied by constant-volume porosimetry is of the order of 1 cm^3 which, assuming a rock with a grain size of $\simeq 100$ μm, gives one a porous medium with up to 1×10^6 individual grains/pores. The simulations were therefore performed on grids of comparable size (128^3). The statistical data were based on a minimum of 1000 runs at this size. When comparing correlated and uncorrelated systems, the pore throat distribution is the *same*. From the generation of the FBM distribution we derive a distribution of zero mean and a variance of 1. This is transformed by $0.5[1+\tanh(f)]$ to put it into the range (0,1). The uncorrelated grids are constructed by randomly choosing values from the same distribution generated by FBM. Choosing throat radii from the same distribution insures that any differences in the simulated curves is due solely to the presence of correlations.

Figure 2 shows the simulated rate-controlled capillary pressure curves for correlated and uncorrelated systems.

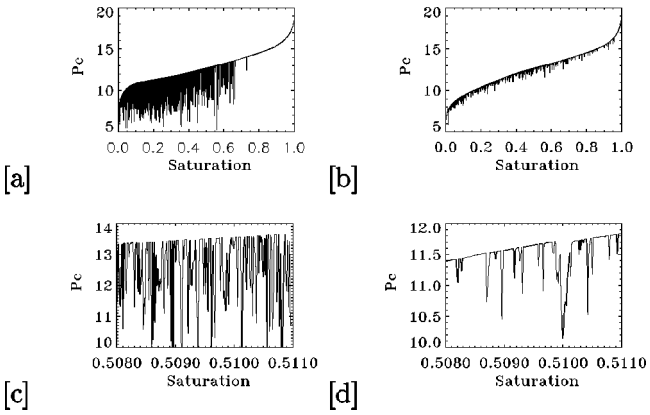

FIG. 2. Volume controlled capillary pressure curves for (a) and (c) uncorrelated, and (b) and (d) FBM grid $\ell_c\to\infty$. (a) and (b) give the curves for the full saturation range, and (c) and (d) for a small range of saturation. The signature of the curves is distinct in both cases. (a) and (c) give no resemblance to the data in Fig. 1.

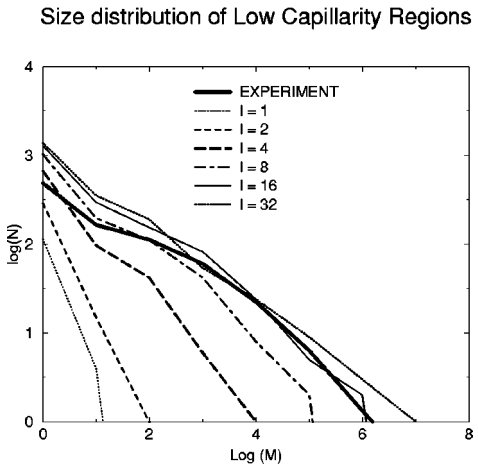

FIG. 3. Size distribution of the low capillarity regions over the saturation range of 60–80%. N is the number of low capillarity jumps measured, and M is the size (number of pores) of the jumps.

Qualitatively the curves are distinctly different. The uncorrelated curves show a higher frequency of jumps in capillary pressure and the jumps have a consistent baseline over the whole saturation range. In contrast, the porosimetry curve for correlated fields exhibits a lower frequency of jumps, is characterized by a more gradual rise in the envelope of the curve, and the baseline of the jumps in the capillary pressure steadily increases with pressure. Comparing Fig. 2 with the experimental curve in Fig. 1 shows that the correlated systems give a better qualitative match, while the uncorrelated case displays no resemblance to the experimental data. This qualitative comparison of the detailed geometry of simulated capillary pressure curves points to the existence of correlated heterogeneity in Berea sandstone.

To evaluate the appropriate length scale ℓ_c of the correlations we consider a quantitative measure used by Yuan and Swanson [14] to characterize the porous rocks: the size distribution of regions of low capillarity over different pressure ranges. The regions of low capillarity measured by constant volume porosimetry can range in size from 1–1000 nl, from a single pore volume to hundreds of pore volumes. At low saturations numerous jumps in the capillary pressure curve of various sizes are noted. At higher saturations the number of jumps into regions of low capillarity are less frequent [compare Figs. 1(b) and 1(d)], although large regions of low capillarity are still invaded at high saturations [Fig. 1(d)]. We have measured the size distribution of low capillarity regions on several Berea sandstone samples in our laboratory. We use this measure to obtain a quantitative prediction of the extent of the length scale ℓ_c of the correlated heterogeneity. At lower saturations differences between the predicted size distributions for varying ℓ_c are difficult to discern. At higher saturations differences between the models becomes more evident. In the uncorrelated case, $\ell_c=1$, for saturations above 60% *no* regions of low capillarity are evident [see Fig. 2(a)]. This contradicts the experimental data shown in Fig. 1. We plot the size distribution of low-capillarity regions in Fig. 3 for models with varying ℓ_c and compare to experiment. It is clear from this plot that the best fit to the experimental data is consistent with an ℓ_c of ten or more pores.

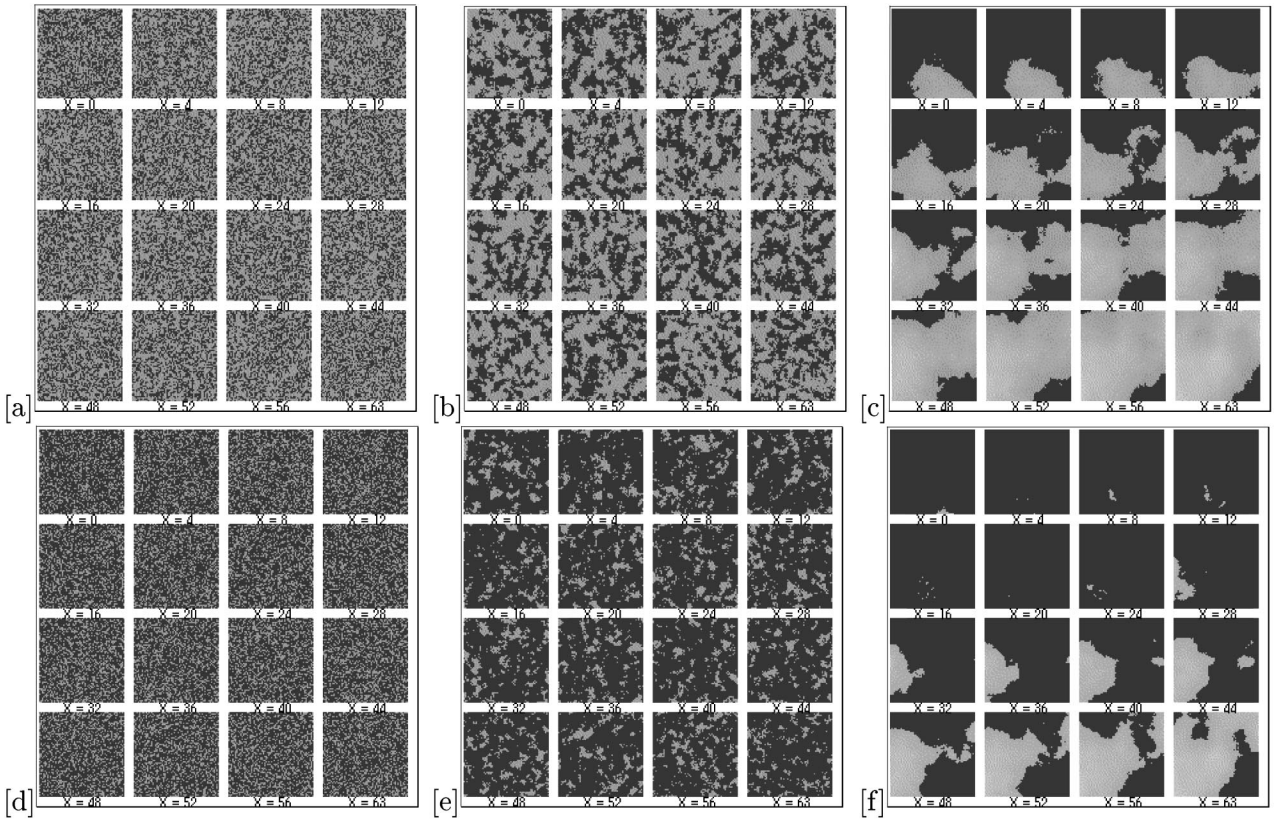

FIG. 4. Number of slices through a 3D 64^3 simulation illustrating the distribution of the nonwetting phase in network simulations after (a)–(c) 25% and (d)–(f) 75% saturation. (a) and (d) $\ell_c=1$; (b) and (e) $\ell_c=8$; (c) and (f) $\ell_c\rightarrow\infty$.

More direct evidence of the presence of correlation at the pore scale comes from the experimental work of Swanson [18]. Swanson presented micrographs of the spatial distribution of a non-wetting phase in a range of reservoir rocks including Berea sandstone. He showed that appreciable portions of the rock are still not invaded by the nonwetting phase at low to moderate nonwetting phase saturations. A micrograph of Berea sandstone at 22% saturation showed large unswept regions of more than 2 mm in extent. Assuming a grain size of 100 μm, uninvaded regions of this extent would contain *thousands* of pores. The experiments of Swanson showed, however, that at higher saturations $>50\%$ the extent of the uninvaded regions is significantly smaller than those observed at lower saturations.

We visualize the distribution of the nonwetting phase during drainage and find that the experimental observation of Swanson can be accounted for if the pore space is correlated with a cutoff length ℓ_c of approximately ten pores. We show in Figs. 4(a)–4(c) a simulation of a displacement for uncorrelated and correlated grids at 25% saturation. The morphology of the displacement on the uncorrelated grid spans much of the lattice and has invaded the majority of the pore space. No large unswept regions are evident. In the two correlated cases, large regions of the pore space remain untouched by the invading fluid in agreement with the observations of Swanson. In Figs. 4(d)–4(f) a simulation of a displacement for uncorrelated and correlated grids at 75% saturation is shown. In our simulation for the FBM grid $\ell_c \to \infty$ [Fig. 4(f)] the regions of the grid uninvaded by the nonwetting fluid remain large. The observations of Swanson are consistent with the simulation in both cases for the cutoff grid [Figs. 4(b) and 4(e)].

Our results suggest that correlated heterogeneity exists down to the pore scale even in a rock like Berea sandstone, which is generally considered to be homogeneous and to exhibit no correlations in its pore size distribution. Moreover, our direct comparison of experiments on Berea sandstone and simulation of porosimetry on correlated grids provides compelling evidence that correlations *do* persist beyond one or two pore lengths, and is quite extended. Most of the previous studies of fluid displacement in porous media [1,2] have used network models of porous media with uncorrelated properties, and random and invasion percolation models have been used for modelling the displacement processes. Our results suggest that the use of random percolation concepts to derive the pore size distribution from mercury porosimetry without considering such extended spatial correlations may neglect an essential aspect of the physics of sedimentary rocks and hence yield misleading results. The error so generated is then compounded if such pore size distributions are used to estimate transport coefficients.

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